The Calculus of Structured Finance
CDO Modeling and Pricing

An exposition of the issues in this challenging area and a demonstration of our competence in the space and expertise with structured product valuation techniques.
1. Current Market Background
2. Basel II and Securitisation
3. Valuation and Pricing Techniques
4. Market Prices and Credit Default Swap Indices
5. REvolution Computing
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Issuance of Structured Products has all but ceased since Q2 ‘07

The Central Banks have become the “counterparty of last resort”. Having restored basic financial stability the EU Member States now have to deal with the impaired assets.
US inter-bank risk premium and declines in US asset-backed commercial paper (a) (b)

Sources: Bloomberg, Board of Governors of the Federal Reserve and Bank calculations.
(a) Blue line shows three-month dollar Libor spread over three-month OIS rate. Magenta line shows five-week rolling average of week-on-week changes in US ABCP outstanding.
(b) Data to close of business on 22 April 2008.
Securitisation exposures are defined as credit exposures created by repackaging the cash flows from a pool of assets into various tranches or asset-backing securities.

The objective of Basel II, uniquely with regard to securitisation, is to bring us closer to risk adjusted capital than we have ever been. The basic premise is that the SFA (Supervisory Formula) will calculate the amount of credit enhancement that the regulators believe is sufficient to cover the underlying credit risk of a given pool of assets.

This level is then compared with the credit enhancement in the transaction. Transactions that do not have a comfortable margin of excess credit enhancement over the minimum required will be subject to a significant capital requirement.
The Supervisory Formula Approach (SFA) is a direct deployment of the Gordy Jones model of “Capital Allocations in securitizations under Uncertainty”. The Gordy Jones (GJ) model, initiated by the requirements of the second Basel Accord, is an academic econometric model developed through the interaction between the ratings agencies and the academic community. It is a relatively complex econometric model.

In securitisation it’s different. At the baseline, the Economic (Risk) Capital and Regulatory Capital numbers are identical (they have been conflated methodologically). The challenge in securitisation is to provide stressed measures of Economic Capital for securitisation issuance (and investment).
The (real world) complexity of Structured Finance

- Pooling and tranching, while being key sources of value in structured finance, are also the main factors behind what might be called the “complexity” of these instruments.
- As far as pooling is concerned, evaluation of risk and return of a structured finance security necessitates modelling the loss distribution of the underlying asset pool, which may be complicated when the pool consists of a small number of heterogeneous assets.
- However, as tranching adds an extra layer of analytical complexity, the evaluation of a structured finance instrument (in other words, a tranche) cannot be confined to analysing asset pool loss. It is also necessary to model the distribution of cash flows from the asset pool to the tranches, i.e. to evaluate the deal’s specific structural features.

- These features, defined via covenants, may entail sets of rules for the allocation of principal and interest payments received from the collateral pool and for the redirection of these cash flows in the case of stress situations, in addition to specifying the rights and duties of various third parties to the transaction.
From a risk management perspective, the most important factor affecting the performance of a CDO deal is the total loss in collateral portfolio value over the life of the deal due to correlated defaults among the collateral.

Each tranche can withstand a characteristic level of loss on the collateral pool before it does not receive its promised interest and principal payments. Performance is also greatly affected by the timing of the defaults, particularly for the equity tranche.

Other risk factors are interest rates, maturing and prepayment rates of collateral, and recovery rates on defaulted collateral. There is also price risk, i.e. the change in value of the collateral due to changes in credit quality or interest rates, and the related issue of reinvestment risk.
# Financial (Economic) Risk Overview (Basel II)

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<td>DURATION LIBOR, Fed Funds, Bank Offered Rate</td>
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<td>Interbank Rate Risk</td>
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<td>Market</td>
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<td>MLR Market Depth</td>
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<td>Funding</td>
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<td>MLR Deposits, Other Funding, Interbank, Market Volatility</td>
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The Asymptotic Single Risk-Factor framework (ASRF)

Gordy describes the methodological outlook of the ASRF as recognising a source of uncertainty reflecting the potential gap between the accounting representation of the tranche (i.e. its position and thickness relative to other holders of principal) and its vulnerability to economic loss.

“The details of the contractual cash flow waterfall are material but unobservable parameters in the true model of the securitization”. From the perspective of the econometrician (in our case, the regulator), such parameters act as sources of random error that must be “integrated out” rather than ignored”.

Gordy introduces one other crucial assumption to ASRF as a premise for the Uncertainty in Loss Provision (ULP) method of appraising economic capital in securitisation. This crucial assumption is fundamental to this approach (the SFA) to this question:

- “It is assumed that the bank's credit portfolio is infinitely fine-grained in the sense that any single obligor represents a negligible share of the portfolio's total exposure.”

The SFA is an implementation of the ULP model. As Gordy remarks: from an operational perspective, the only significant challenge in implementing the ULP model lies in obtaining the pool characteristics $n$, $Kirb$ and $ELGD$; therefore securitisation transactions may differ in legal form but they do not differ in risk.
Collateralised debt obligations are asset-backed securities (ABS) based on two main debt types:
- Collateralised bond obligations (CBO)
- Collateralised loan obligations (CLO)

CDOs are pools of assets made up of credit derivatives and having possibly complex cash-flow structure
Steps in Structured Product Modeling

- Model the cash-flow of the structure (subordinate structures, fees and hedge premiums, principal and interest waterfalls, coverage tests, credit enhancements and over-collateralisation, triggers)
- Model underlying portfolio by covering multi-year horizons (for longer maturities), factor modelling to account for industry/country diversification, and modelling short term interest rates (for capturing interest rate risk)
- Monte Carlo pricing simulations
- Loan amortization (schemes)
- Depreciation (fixed declining, sum of year's digits, straight-line...)
- Present value
- Future value
- Find internal rate of return
- Convert nominal rate to effective rate
- Convert effective rate to nominal rate
- Cash flow duration (macaulay, modified, effective etc.)
- Price bond
- Price future
- Payment
- Cash flow convexity
In CDO pricing, default is typically modelled in the double stochastic framework with random time and random hazard rate, assumed to be functions of a multidimensional state process.

Well known pricing models are:

- Vasicek (Ornstein-Uhlenbeck)
- Cox-Ingersoll-Ross (CIR)
- Heath-Jarrow and Morton (HJM) Framework
- Longstaff and Schwarz
- Hull & White
- Duffie and Singleton
- Copula

The yields to maturity on default free pure discount bonds and the instantaneous interest rate are modelled as affine functions of the latent state variable.
(Example) Estimation Techniques

- Multivariate Ornstein-Uhlenbeck
- Vasicek State-Space Representation
- Kalman Filter
- Multivariate Cox-Ingersoll-Ross
- Copula

- See Asymptotix Greenpaper for a fuller exposition
Technique Comparison - Hazard Profile

Multivariate Ornstein-Uhlenbeck

Multivariate Cox-Ingersoll-Ross
Market-implied expectations of ultimate loss rates on US sub-prime mortgages (a) (b)

- Source: Bank calculations using data from JPMorgan Chase & Co.

- (a) Based on the collateralised debt obligation (CDO) model used in ‘A simple CDO valuation model’, Bank of England Financial Stability Review, Box 1, December 2005, pages 105–06, applied to 2007 H1 ABX tranches, assuming these prices reflect only credit risk.

- (b) The model estimates a market-implied probability of default of the underlying mortgages. This is a ‘risk-neutral’ default probability. In the likely case that investors are averse to risk, the perceived probability of default will be lower than under the risk-neutral measure.
Recently Factor Models have been developed to take account of market prices as key indicators of risk. There is some controversy here since this approach relies upon the seemingly slightly tenuous idea that the market can price risk in a financial institution or a corporate from an external perspective when only that institution’s private and confidential internal data should be able to support accurately and up to date prices of its risk. “Do they know something we don’t?!”

On the basis of Credit Default Swap (CDS) prices (spreads in particular); repeated empirical testing of this proposition (generally expressed in factor models) has demonstrated its efficacy and validity and many theoretic explanations have been presented in terms of the Factor Model’s latent variable. The data, the ‘fit’ and the explanation all seem to add up and there is movement towards general consensus that the CDS price (both index and stock-specific) is becoming a common shared indicator of credit risk. Significant market developments in loan pricing predicated upon CDS prices rather than LIBOR, consequent upon the failure of the latter in the CC are good evidence of this and further evidence of the application of quantitative techniques in mainstream financial transactions.

The proposition is that statistical modelling (using the Factor Model) is possible based upon multiple default indicators (CDS spreads) to drive out the latent variable scores, thus quantifying risk economic capital. This type of factor analytic is typical in other areas of applied statistical modelling that use multiple observable indicators of the true (latent) endogenous variable (risk).

www.asymptotix.eu/ecap.pdf
Major UK banks’ and LCFIs’ credit default swap premia (a) (b)

- Sources: Markit Group Limited, Thomson Datastream, published accounts and Bank calculations.
- (a) Data to close of business on 20 October 2008.
- (b) Asset-weighted average five-year premia.
- (c) April 2008 Report.
European and US speculative-grade corporate default rates and forecasts (a) (b)

- Source: Moody’s Investors Service.
- (a) Trailing twelve-month issuer-weighted speculative-grade corporate default rates.
- (b) Solid lines show historical data. Dashed lines show Moody’s forecasts for October 2008 to September 2009.
1. Most sub-prime mortgages and many other forms of risky debt were purchased in CDOs.

2. Senior tranches were rated AAA and were considered to have virtually no risk.
   - Our ability to structure financial products ran ahead of our understanding of the risk
   - The new forms of financing such as CLOs and CDOs allowed distressed firms to postpone default by bringing forth liquidity which ceteris paribus would not have existed
   - Corporate default rates were unusually low in the second half of the 2000s, financial innovation was partly responsible for the low default rates

3. Thus extant structured assets or re-structured portfolios of ‘seasoned’ exposures (pre-2007) need to be tested with the most rigorous toolsets available from a ‘Fair Value’ standpoint.
   - Default is associated with extreme events
   - Defaults are driven by common risk factors
     - Common negative shocks to cash flows
     - Conditional on these common factors, defaults are independent (idiosyncratic)
   - Default of one counterparty causes financial distress to other counterparties with whom it is related.
The Issue in a Nutshell is Statistical

1. Default correlations are the most important drivers of the tails of portfolio credit risk distributions
   - The correlation of defaults depends on the lower tail dependence of the joint distribution
2. The only way to estimate correlation or covariance is statistically, the only language in which to present them is mathematics.
   - Default correlations cannot be measured directly, and must be inferred from a model
3. In a one factor model with Asymmetric GARCH factor, multi-period returns will have lower tail dependence and higher default correlations
   - This entails that even senior tranches of a CDO are more risky and makes them sensitive to changing volatilities.
4. Without considering how the individual exposures in a structured instrument are inter-related the structured product (constructed of many inter-related exposures) cannot be priced or valued fairly in any logical or valid manner.
Use of specialists may be required when an auditor may not have the necessary skill and knowledge in valuation concerns i.e. where fair valuation involves:

- Significant use of unobservable inputs
- Complexity of the valuation technique
- Materiality of the fair value measurement

Using the work of a specialist:

- Obtain understanding of the methods and assumptions used
- Make appropriate tests of data
- Evaluate whether the specialist’s findings support the assertions in the financial statements

Use of pricing services:

- Determine nature of information provided
- Active market, observable inputs, mark to a model
- Where and When are non-market input data applicable and reliable
- Price based on principal or most advantageous market
- Is the price provided by the service realizable?

Adjust audit procedures accordingly including gaining an understanding of the input assumptions
Overview
Why REvolution R?

Scale on Windows –
Windows 64 bit enabled
HPC Server 2008 enabled

Integration with Windows Tools –
• Including Excel and Visual Studio IDE

• Rapidly go from prototyping to production -
  • Multithreaded, highly optimized on desktop
  • HPC Server 2008 parallel heterogeneous cluster support
REvolution R in Finance

• Quants love it
  • Powerful time-series objects, graphics and methods
  • Superb data connectivity (Reuters, Bloomberg, etc.)
  • Scale technical models across PCs and clusters to avoid recoding

• Risk Analysis is a growing market
  • Sophisticated, state of the art methods
  • Programs scale from exploratory data analysis to large-scale production problems without recoding
Financial Risk  (Bank of England Fanchart)
If one is engaged in a process to consider the fair or market or ‘economic’ value of Structured Products (Collateralized Debt Obligations (CDOs) etc) which is the challenge of the moment, right now; then one has to start from a reliable (well researched) theoretical standpoint. These ‘things’ are ‘tough customers’!

In addition all the evidence points to the necessity of the most advanced predictive analytic computing power being at your disposal coupled with the most tractable and yet disaggregated data management. After all is this not just what the banks did not deploy before the crisis?

These ‘structured assets’ are by definition ‘Level 3’ in IFRS7 terms and thus the valuation philosophy of ‘mark to model’ must apply since no active reference market for such securities exists in anyway globally. The question is what is the optimum model? This has to be a model defined by practitioners (academic, supervisory or market participants) & in the public domain.

Little is known about how and why spreads of asset-backed securities are influenced by loan tranche characteristics. Default and recovery risk characteristics represent the most important group in explaining loan spread variability. Marketability explains a significant portion of the spreads’ variability but that factor is irrelevant to a current model, most of the common pricing characteristics between ABS, MBS and CDO differ significantly. Furthermore, applying the same pricing estimation model to each security class revealed that most of the common pricing characteristics associated with these classes have a different impact on the primary market spread exhibited by the value of the coefficients.

The predominant industry approach to pricing and hedging CDOs and tranched index products is known as the “copula.” The version of the copula model most commonly used for quotation purposes, is known as the “base correlation.” A recent article on the Copula in Wired magazine, was interesting in its depth (it had none!). The article was heavily commented upon but it reflects a zero level understanding of the Gaussian Copula which then maybe reflects why “the wizards of wall street” got it so drastically wrong, its nothing to do with the technique, its just that they did not understand 1) what it is for and 2) how it does what it is for.
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2\textsuperscript{nd} presentation EUEC DG COMP